- Yu. V. Lapin, Turbulent Boundary Layer in Supersonic Gas Flow [in Russian], Nauka, Moscow (1970).
- 21. Yu. V. Baryshev, Yu. A. Vinogradov, A. I. Leont'ev, and V. I. Rozhdestvenskii, "Recovery factors on a permeable surface and in the gas curtain region in a supersonic turbulent boundary layer," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, 2, 131 (1972).
- 22. E. R. Bartle and B. M. Leadon, "Experimental evaluation of heat transfer with transpiration cooling in a turbulent boundary layer at M = 3.2," J. Aero. Sci., <u>27</u>, No. 1, 1125 (1960).
- A. I. Leont'ev, B. P. Mironov, and A. V. Fafurin, "Investigation of turbulent transfer in the initial section of a porous pipe under isothermal conditions," Teplofiz. Vys. Temp., No. 6, 1134 (1960).
   C. G. Papas, "Effect of injection gases on the skin friction and heat transfer of the
- 24. C. C. Papas, "Effect of injection gases on the skin friction and heat transfer of the turbulent boundary layer," J. Aero. Sci. Rep. N 59-78 (1959), p. 21.

INTENSIFICATION OF CONVECTIVE HEAT EXCHANGE IN CHANNELS WITH A POROUS HIGH-THERMAL-CONDUCTIVITY FILLER. HEAT EXCHANGE WITH LOCAL THERMAL EQUILIBRIUM INSIDE THE PERMEABLE MATRIX

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Results are presented from analytical and experimental studies of intensification of convective heat exchange in a channel with a porous high-thermal-conductivity filler in the case of moderate external heating.

The placement, in a channel, of a porous, high-heat-conducting material which is strongly bound to the channel walls causes a qualitative change in the mechanism and an intensification of heat transfer; heat is transferred from the channel walls by conduction through the framework inside the permeable matrix and is thence diffused in the flow as a result of intrapore heat exchange. The obvious physical concept behind this method was the reason that the development of a technology for making porous metals was accompanied by the proposal [1-5] of a large number of designs of various heat exchangers in which either the channels or the intertube space is filled with a permeable metal. Later the phenomenon of a substantial intensification of heat exchange was confirmed experimentally [6-8]. In particular, as a result of cooling provided by pumping water through a porous base, reliable operation of a laser reflector was realized at a thermal load  $q_W = 8 \cdot 10^7 \text{ W/m}^2$  in [8]. Theoretical study of the process was held up for a long time by the absence of necessary information on the properties of permeable matrices. Recently, as data on the structural, hydraulic [9], heatexchange [10], and heat conduction [11] characteristics of different porous metals has been accumulated and generalized, there has been a rapid increase in the number of publications with analytical results [12-19]. However, not all of these works are of a qualitative, formulative nature and do not offer an exhaustive evaluation of the effect of different parameters on intensification of heat exchange in the process in question.

Formulation of the Problem. A channel of constant cross section (Fig. 1.1) of width or diameter  $\delta$  is filled with a porous high-thermal-conductivity material beginning with the section z = 0. A single-phase heat carrier flows through the channel. The section z = 0coincides with the beginning of the external heating of the walls, which is the same on both sides of the channel. The permeable matrix has perfect thermal and mechanical contact with the walls, is isotropic, and has a thermal conductivity  $\lambda$  which is the same in all directions. The thermal conductivity of the heat carrier  $\lambda_0$  is small compared to  $\lambda$  (which is determined by the very essence of the method) and its thermophysical properties are constant.

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Fig. I.1. Physical model of the process.

Thus, the mass rate remains constant across the channel G = const. The rate  $h_V$  of volumetric intrapore heat exchange is great but nonetheless finite [10]. Consequently, beginning with a certain level of the external heat flow introduced into the channel wall, the temperature difference T - t between the porous material and the heat carrier becomes appreciable and increases constantly.

Given these assumptions, the temperature field of the permeable matrix T and the heat carrier t in the plane channel is described by the system of equations

$$\lambda\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2}\right) = h_{\nu}(T-t); \tag{I.1}$$

$$Gc \frac{\partial t}{\partial z} = h_v (T - t). \tag{1.2}$$

The value of  $h_v$  remains constant with a constant mass rate G [10].

Five boundary conditions must be prescribed for system (I.1)-(I.2). Three of them are independent of the character of the external heating and reflect the following features of the process. There is no heat transfer from the porous material counter to the incoming flow because the thermal conductivity of the latter is negligibly small:  $\lambda \frac{\partial T}{\partial z}|_{z=0} = 0$ . The temperature of the heat carrier at the inlet to the permeable matrix remains constant  $t|_{z=0} = t_0$ . The symmetry condition  $\frac{\partial T}{\partial y}|_{y=0} = 0$  is satisfied on the channel axis. The remaining two boundary conditions are determined by the external heating.

If the channel walls are washed by an external flow with a constant temperature  $t_{\infty}$  and if the rate of convective heat exchange  $\alpha_{\infty}$  on both sides is constant (boundary conditions of the third kind), then as the heat carrier moves through the porous material its temperature approaches  $t_{\infty}$ . Using the dimensionless quantities:

$$\vartheta = \frac{t - t_{\infty}}{t_0 - t_{\infty}}; \ \Theta = \frac{T - t_{\infty}}{t_0 - t_{\infty}}; \ \xi = z/\delta; \ \zeta = y/\delta,$$
(1.3)

we can write the boundary conditions for this case as follows:

$$\boldsymbol{\xi} = 0, \ \frac{\partial \Theta}{\partial \boldsymbol{\xi}} = 0; \tag{I.4}$$

$$\boldsymbol{\xi} = \boldsymbol{0}, \ \boldsymbol{\vartheta} = \boldsymbol{1}; \tag{I.5}$$

$$\zeta = 0, \quad \frac{\partial \Theta}{\partial \zeta} = 0; \tag{1.6}$$

$$\zeta = 1/2, \ \Theta = -\frac{1}{\text{Bi}} \frac{\partial \Theta}{\partial t}; \qquad (1.7)$$

$$\xi \to \infty, \ \vartheta \to 0.$$
 (I.8)

The heat-transfer coefficient in the criterion  $Bi = k_{\infty} \delta/\lambda$  includes the heat-transfer resistance  $\delta_W/\lambda_W$  of the wall:  $k_{\infty} = (1/\alpha_{\infty} + \delta_W/\lambda_W)^{-1}$ 

With a high rate of external heating (Bi  $\rightarrow \infty$ ), Eq. (I.7) changes into the condition of constancy of the temperature of the channel wall  $T_W = T_\infty$ :

$$\zeta = 1/2, \ \Theta = 0.$$
 (I.9)

Equations (I.4)-(I.6), (I.8), and (I.9) represent boundary conditions of the first kind.

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When the channel walls are acted upon by a constant external heat flow  $q_W$ , the mean temperature of the heat carrier t at the outlet of the permeable matrix is proportional to its length. If we express the dimensionless temperatures as follows for this variant

$$\vartheta = \frac{t - t_0}{q_w \delta/\lambda}; \quad \Theta = \frac{T - t_0}{q_w \delta/\lambda},$$
(I.10)

then boundary conditions (I.4), (I.6) remain the same but (I.5), (I.7), and (I.8) are replaced by

$$\zeta = 1/2, \ \frac{\partial \Theta}{\partial \zeta} = 1; \tag{I.11}$$

$$\boldsymbol{\xi} = 0, \ \boldsymbol{\vartheta} = 0 \ \boldsymbol{\xi} \to \infty, \ \boldsymbol{\overline{\vartheta}} = \frac{\overline{t} - t_0}{q_w \delta/\lambda} = \frac{2}{\text{Pe}} \boldsymbol{\xi}. \tag{1.12}$$

With allowance for (I.3) or (I.10), Eqs. (I.1) and (I.2) take the form

$$\frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \xi^2} = \gamma^2 (\Theta - \vartheta); \qquad (I.13)$$

$$\operatorname{Pe}\frac{\partial \vartheta}{\partial \xi} = \gamma^2 (\Theta - \vartheta). \tag{I.14}$$

It should be emphasized that only the thermal conductivity of the porous material  $\lambda$  and not that of the heat carrier  $\lambda_0$  is used in the mathematical formulation of the problem (I.1)-(I.14). Thus, the thermal conductivity of the permeable filler also goes into the determining parameters Be, Pe, and  $\gamma^2$  (and, as will be shown below, in the criterion Nu). The parameter Pe =  $G\delta c/\lambda$  is the analog of the Peclet flow criterion and represents the ratio of the quantities of heat transferred along the channel by the heat carrier and, through conduction, by the porous material. The dimensionless parameter  $\gamma^2 = h_V \delta^2/\lambda$  characterizes the rate of volumetric intrapore heat transfer. Both of the parameters Pe and  $\gamma^2$  are constant with a constant mass rate G across the channel.

For a circular channel of diameter  $\delta$ , all of the equations (I.3)-(I.14) remain the same except for the replacement of (I.13) by

$$\frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \zeta^2} + \frac{1}{\zeta} \frac{\partial \Theta}{\partial \zeta} = \gamma^2 \left(\Theta - \vartheta\right). \tag{I.15}$$

System (I.13)-(I.14), with the stated boundary conditions, can be solved in analytical form by the method of separation of variables. For example, with boundary conditions (I.4)-(I.8), having expressed  $\mathcal{S}$  in the form of the product  $\vartheta(\xi, \zeta) = \varphi(\zeta)\psi(\xi)$ , we can use (I.13)-(I.14) to obtain the following system of ordinary differential equations

$$\varphi'' + 4\mu^2 \varphi = 0; (I.16)$$

$$\frac{Pe}{\gamma^2} \psi''' + \psi' - Pe\left(1 + \frac{4\mu^2}{\gamma^2}\right)\psi' - 4\mu^2\psi = 0.$$
 (1.17)

It should be emphasized that the form of the last equation of (I.17) does not depend on the channel geometry. The solution of the problem is then written as follows:

$$\vartheta(\zeta, \xi) = \sum_{1}^{\infty} A_n \varphi_n (2\mu_n \zeta) [\exp(\varepsilon_{1n} \xi) + C_{1n} \exp(\varepsilon_{2n} \xi) + C_{2n} \exp(\varepsilon_{3n} \xi)],$$

where  $\varepsilon_{1n}$ ,  $\varepsilon_{2n}$ ,  $\varepsilon_{3n}$  are the roots of a third-order characteristic equation

$$\frac{\mathrm{Pe}}{\gamma^2} \varepsilon^3 + \varepsilon^2 - \mathrm{Pe} \left(1 + 4\mu_n^2/\gamma^2\right)\varepsilon - 4\mu_n^2 = 0$$

in which  $\mu_n$  are characteristic values of the problem.

It is hard in the general case to evaluate the effect of the parameters Pe and  $\gamma^2$  on the roots of this equation and the solution of the problem as a whole with variable values of  $\mu_n$ , dependent on the boundary conditions and channel geometry. Thus, let us concern ourselves primarily with several special cases of the process in question, when the roots of the last equation can be expressed in simple form. All of these special cases permit simplification of Eq. (I.17).



Fig. I.2. Effect of the parameter Pe on the change in local (1-4) and mean (1'-4') heattransfer criteria in the inlet section of a permeable matrix in a plane channel with a constant wall temperature ( $T_W =$  $t_{\infty}$ , Bi  $\rightarrow \infty$ ): 1, 1') Pe  $\rightarrow \infty$ ; 2, 2') Pe = 100; 3, 3') 10; 4, 4') 1; 5) local Nu for a flow with a parabolic velocity profile in a channel without a filler and without allowance for the effect of axial heat conduction.

Heat Exchange with Local Thermal Equilibrium Inside the Porous Material. With a moderate external heat flow, the temperatures of the permeable matrix and the heat carrier do not differ appreciably due to the high rate of intrapore heat transfer. Thus, we have local thermal equilibrium inside the porous structure: T = t. We will henceforth exactly determine the conditions under which this assumption is valid.

Assuming that  $\vartheta = \Theta$  (or that  $\gamma^2 \rightarrow \infty$ ), system (I.13)-(I.14) can be written as a single equation

$$\frac{\partial^2 \vartheta}{\partial \xi^2} + \frac{\partial^2 \vartheta}{\partial \zeta^2} - \operatorname{Pe} \frac{\partial \vartheta}{\partial \xi} = 0, \qquad (1.18)$$

Eq. (I.16), for determining the function  $\varphi(\zeta)$  remains the same, while Eq. (I.17) is simplified:

$$\psi'' - \operatorname{Pe} \psi' - 4\mu^2 \psi = 0.$$

The number of boundary conditions is reduced to four - condition (I.4) is eliminated.

Boundary Conditions of the First and Third Kinds. With boundary conditions (I.5)-(I.8), the solution of Eq. (I.18) has the form

$$\vartheta = \Theta = 2 \sum_{1}^{\infty} \frac{A_n \mu_n}{\sin \mu_n} \cos (2\mu_n \zeta) \exp (-B_n \xi).$$
(1.19)

Here, we used the notation

$$A_n = \frac{1}{\mu_n} \left( \frac{\sin^2 \mu_n}{\mu_n + \sin \mu_n \cos \mu_n} \right); \tag{I.20}$$

$$B_n = [(Pe/2)^2 + 4\mu_n^2]^{1/2} - Pe/2, \qquad (1.21)$$

where  $\mu_{\textbf{n}}$  are characteristic values satisfying the familiar characteristic equation

$$\mu \, \text{tg} \, \mu = \text{Bi}/2.$$
 (1.22)

With a constant channel-wall temperature  $T_W = t_{\infty}(Bi \rightarrow \infty)$ , we have  $\mu_n = (2n - 1)\pi/2$ , n = 1, 2, 3, ...,  $A_n = 1/\mu_n^2$ .

The complete local criterion Nuk, determining the heat-transfer rate  $k = ((1/\alpha + 1)k_{\infty})^{-1}$  between the heat carrier inside the porous material filling the channel and the external flow, is calculated from the equation



Fig. I.3. Dependence of heat-transfer criteria  $Nu_{k\infty}$  and  $Nu_{\infty}$  for the region of stabilized heat transfer on the rate of external heat transfer: 1)  $Nu_{k\infty}$ , plane channel; 2)  $Nu_{k\infty}$ , circular channel; 3)  $Nu_{\infty}$ , plane channel; 4)  $Nu_{\infty}$ , circular channel.

$$\mathrm{Nu}_{k} = k\delta/\lambda = -\frac{1}{\overline{\vartheta}} \left. \frac{\partial \Theta}{\partial \zeta} \right|_{\zeta = 1/2} = -\frac{4}{\overline{\vartheta}} \sum_{1}^{\infty} A_{n} \mu_{n}^{2} \exp\left(-B_{n} \xi\right).$$
(1.23)

This criterion also includes the thermal conductivity  $\lambda$  of the permeable filler.

The mean temperature  $\vartheta$  of the heat carrier over the cross section

$$\overline{\vartheta} = 2 \int_{0}^{1/2} \vartheta d\zeta = 2 \sum_{n=1}^{\infty} A_n \exp\left(-B_n \xi\right).$$
 (I.24)

The local criterion Nu, determining the heat-transfer rate  $\alpha$  between the flow in the channel and the channel wall, is determined by the following relation after Nu<sub>k</sub> is calculated

$$Nu = \alpha \delta / \lambda = Bi Nu_{b} / (Bi - Nu_{b}).$$
 (1.25)

With a constant wall temperature (Bi  $\rightarrow \infty$ ), we have Nu<sub>k</sub>  $\rightarrow$  Nu.

It follows from the above results (I.19) that the change in the temperatures of the porous material and the heat carrier along the channel depend separately on the coordinate  $\xi$  and the parameter Pe. With an increase in Pe, the following asymptote is realized

$$B_n \xi|_{\mathsf{Perm}} \to 4\mu_n \xi/\mathsf{Pe},$$
 (I.26)

eliminating the effect of axial heat conduction. This is equivalent to dropping the first term in the left side of Eq. (I.1):  $\lambda \partial^2 T/\partial z^2 = 0$ . In this case, the change in the temperature of the heat carrier along channels of any form depends only on one parameter  $\xi/Pe$ .

Figure 1.2 shows the effect of the parameter Pe on the rate of local heat transfer with a constant wall temperature  $(Bi \rightarrow \infty)$ . Certain features should be pointed out. For cases in which no allowance is made for axial heat conduction (Pe  $\rightarrow \infty$ , curves 1 and 5), in the transition to the "fuller" uniform velocity profile the heat-transfer rate increases both on the initial section and in the region of stabilized heat transfer. Dependence 2 for Pe = 100 nearly coincides with the first dependence, i.e., at Pe > 100 the effect of axial heat conduction can be ignored. All of the values of Pe with a uniform velocity profile (curves 1-4) correspond to the same limiting value of Nu $\infty$  in the region of stable heat exchange. Lengthwise conductive heat transfer (at Pe < 100) increases both the heat-transfer rate on the inlet section and the length of this zone.

We must also note the following important property — the heat-transfer rate in a channel with a porous filler is determined by the value of Pe but does not depend separately on the Reynolds number Re in the channel, i.e., the flow regime (laminar or turbulent) has no effect, in contrast to the case in hollow channels.

The dashed lines 1'-4' in Fig. I.2 show the change in the mean heat-transfer criterion Nu<sub>k</sub> for the inlet section of the porous material:

$$\overline{\mathrm{Nu}}_{k} = \frac{1}{\xi} \int_{0}^{\xi} \mathrm{Nu}_{k}(\xi) d\xi.$$
(1.27)

With sufficiently high values of  $\xi$  in the region of stable heat exchange, both the local and the mean heat-transfer criteria Nu<sub>k</sub> and Nu<sub>k</sub> acquire identical constant limiting values Nu<sub>k</sub> $\infty$ . In this case, we can limit ourselves to the first term of the series in (I.23), from which it follows that Nu<sub>k</sub> $\infty = 2\mu_1^2$ . The characteristic values  $\mu_1$  depend only on the parameter Bi. Thus, for a certain channel geometry, the limiting values Nu<sub>k</sub> $\infty$  depend only on Bi (Fig. I.3). With Bi  $\rightarrow \infty$ , we obtain Nu<sub>k</sub> $\infty \rightarrow$  Nu $\infty$ . With Bi  $\rightarrow 0$ , the criterion Nu<sub>k</sub> $\infty$  decreases as well (Nu<sub>k</sub> $\infty \rightarrow 0$ ), since Nu $\infty$  increases.

The length  $\xi_{l}$  of the initial section of thermal stabilization for the local heat-transfer coefficient is usually determined as the distance from the beginning of the external heating (in the present case, from the inlet to the porous material) over which the condition Nuk( $\xi_{l}$ ) = 1.01(Nuk $\infty$ ) is satisfied. From here, with allowance for Eq. (I.23), it follows that:

$$\xi_{l} = \frac{1}{B_{2} - B_{1}} \ln \left( 100 \mu_{2}^{2} A_{2} / \mu_{1}^{2} A_{1} \right).$$
(1.28)

Boundary Conditions of the Second Kind. The solution of (I.18) with boundary conditions (I.6), (I.11)-(I.12) has the form

$$\vartheta = \Theta = \frac{2}{Pe} \xi + \zeta^2 - \frac{1}{12} - \sum_{1}^{\infty} \frac{(-1)^n}{\mu_n^2} \cos(2\mu_n \zeta) \exp(-B_n \xi).$$
(1.29)

Here,  $B_n$  is determined from Eq. (I.21), while the characteristic values  $\mu_n = n\pi$ , n = 1, 2, 3, ... The temperature of the channel wall is found from (I.29) with  $\zeta = 1/2$ :

$$\Theta_{w} = \frac{T_{w} - t_{0}}{q_{w}\delta/\lambda} = \frac{2}{\text{Pe}} \xi + \frac{1}{6} - \sum_{1}^{\infty} \frac{1}{\mu_{n}^{2}} \exp\left(-B_{n}\xi\right).$$
(1.30)

The local heat-transfer coefficient  $\alpha$  for heat transfer from the channel wall to the flow inside the porous material is referred to the difference between the wall temperature and the mean temperature of the heat carrier. In this case, it is determined from the expression

$$\mathrm{Nu} = \alpha \delta / \lambda = (\Theta_w - \overline{\delta})^{-1} = \left[ \frac{1}{6} - \sum_{1}^{\infty} \frac{1}{\mu_n^2} \exp\left(-B_n \xi\right) \right]^{-1}.$$
 (I.31)

The mean heat-transfer coefficient  $\overline{\alpha}$  is calculated from the mean integral temperature difference

$$\overline{\mathrm{Nu}} = \left[\frac{1}{\xi} \int_{0}^{\xi} (\Theta_{w} - \overline{\vartheta}) d\xi\right]^{-1} = \left[\frac{1}{6} - \frac{1}{\xi} \sum_{n=1}^{\infty} \frac{1 - \exp\left(-B_{n}\xi\right)}{\mu_{n}^{2}B_{n}}\right]^{-1}.$$
(I.32)

To calculate the length  $\xi_{\zeta}$  of the initial thermal section, we use Eq. (I.31) to obtain the expression

$$\xi_l = \frac{1}{B_1} \ln (600/\mu_1^2). \tag{I.33}$$

It is interesting to note that we can also obtain an analytical expression to determine the length  $\xi_{\ell}$  of the initial section for the mean heat-transfer coefficient: from (I.32) we find  $\overline{\xi_{\ell}} = 600/B_1\mu_1^2$ . It follows from this that the length ratio  $\overline{\xi_{\ell}}/\xi_{\ell}$  is independent of the parameter Pe and remains constant  $\overline{\xi_{\ell}}/\xi_{\ell} = 14.8$ .

Comparison of the results shown in Figs. I.2 and I.4 illustrates that all of the qualitative features of heat transfer in a channel with a porous filler noted earlier for the process with boundary conditions of the first and third kinds also hold for boundary conditions of the second kind.

It follows from Fig. I.5 that the difference between the results for the cases with and without allowance for axial heat conduction, which is substantial for small Pe, gradually disappears as Pe increases and approaches Pe = 100. Meanwhile, the length of the initial thermal section is greater in the first case due to lengthwise heat transfer along the porous material. Values of the length  $\xi_{7}$  for the leftmost (Pe = 0) and rightmost (Pe  $\rightarrow \infty$ ) points in Fig. I.5 are shown in Table 1.





Fig. I.4. Effect of the parameter Pe on the change in the local (1-4) and mean (1'-4') heat-transfer criteria on the inlet section of a permeable matrix in a plane channel with a constant external heat flow  $(q_w = \text{const})$ : 1, 1') Pe  $\rightarrow \infty$ ; 2, 2') Pe = 100; 3, 3') 10; 4, 4') 1; 5) local Nu for a flow with a parabolic velocity profile in a channel without a filler and without allowance for axial heat conduction.

Fig. 1.5. Dependence of the length of the initial thermal section in a channel with a porous filler on the parameter Pe: 1) plane channel,  $q_W = const; 2)$  circular channel,  $q_W$  = const; 3) plane channel,  $T_W$  = const; 4) circular channel,  $T_w = const; 1'-4'$ ) same, respectively, but without allowance for axial heat conduction  $(\lambda \partial^2 T/\partial z^2 = 0)$ .

Effect of Anisotropy of Thermal Conductivity of the Permeable Matrix. Many metals, such as those used in the form of netting and fibers, have physical properties with a pronounced anisotropy. These properties include thermal conductivity. We will study heat exchange in a a channel with a porous filler (see Fig. I.1) in which thermal conductivity in the transverse  $\lambda_y$  and longitudinal  $\lambda_z$  directions is quite different. Meanwhile,  $\lambda_y > \lambda_z$ . We will compare this with results for a uniform permeable insert with a thermal conductivity  $\lambda_y$  which is the same in all directions. Thus, we will evaluate the effect of a decrease in longitudinal thermal conductivity  $\lambda_z$  when the transverse thermal conductivity  $\lambda_y$  is constant.

The temperature field of a heat carrier and a porous anisotropic filler is described by the following equation when their temperatures are equal T = t

$$\lambda_z \frac{\partial^2 t}{\partial z^2} + \lambda_y \frac{\partial^2 t}{\partial y^2} - Gc \frac{\partial t}{\partial z} = 0.$$
 (1.34)

Let us examine as an example a variant with boundary conditions of the third kind. Using the quantities

$$\Lambda^{2} = \lambda_{y}/\lambda_{z} > 1; \ \zeta = y/\delta; \ \xi_{1} = \xi \Lambda = \frac{z}{\delta} \Lambda;$$
  
$$\theta = \frac{t - t_{\infty}}{t_{0} - t_{\infty}}; \ Pe_{1} = Pe \Lambda; \ Pe = G\delta c/\lambda_{y}; \ Bi = k_{\infty}\delta/\lambda_{y}, \qquad (I.35)$$

we reduce Eq. (I.34) and the boundary conditions to dimensionless form:

$$\frac{\partial^2 \vartheta}{\partial \xi_1^2} + \frac{\partial^2 \vartheta}{\partial \zeta^2} - \operatorname{Pe}_1 \frac{\partial \vartheta}{\partial \xi_1} = 0; \qquad (1.36)$$

$$\frac{\partial \mathbf{\theta}}{\partial \boldsymbol{\zeta}}\Big|_{\boldsymbol{\zeta}=0} = 0; \quad \frac{\partial \mathbf{\theta}}{\partial \boldsymbol{\zeta}}\Big|_{\boldsymbol{\zeta}=1/2} = \frac{-1}{\mathrm{Bi}} \frac{\partial \mathbf{\theta}}{\partial \boldsymbol{\zeta}} .$$

$$(1.37)$$

In such a form, the problem coincides fully with problem (I.18), (I.5)-(I.8) on heat transfer in a channel with a porous isotropic insert with a thermal conductivity  $\lambda = \lambda_y$ . The only difference is that the quantities  $\xi$  and Pe are replaced here by  $\xi_1 = \xi \Lambda$ ; Pe<sub>1</sub> = PeA.

Parameters	Plane cha	nne1	Circular channel			
	T <sub>w</sub> =const	$q_{w} = \text{const}$	$T_w = \text{const}$	$q_{\omega} = \text{const}$		
Nu <sub>∞</sub>	4,93	ō,Ū	5,78	S,0		
$\frac{\xi_l}{\text{Pe}}\Big _{\text{Pe} \to \infty}$	0,0584	0,104	0,0466	0,068		
ξlipe=0	0,733	0,655	0,739	[0,523		

TABLE 1. Main Heat-Transfer Characteristics in Channels with Local Thermal Equilibrium between the Porous Filler and Heat Carrier

TABLE 2. Characteristics of Forous Netted Metallic Inserts in an Annular Channel

Speci- men	Porositv	Meta1	δ, mm	1, <b>mm</b>	<sup>d</sup> intr, mm	<sup>d</sup> extr, mm	<sup>d</sup> thr, µm	1/8
1	0,37	1Kh18N9T	15	42	45	60	150	2,8
2	0,49	1Kh18N9T	15	42	45	60	200	2,8
3	0,55	1Kh18N9T	15	42	45	60	220	2,8
4	0,65	1Kh18N9T	15	42	45	60	250	2,8
5	0,50	Brass	15	42	45	60	—	2,8
6	0,55	1Kh18N9T	15	102	45	60	220	6.8

Thus, with allowance for these changes, it is possible to use all of the results (I.19)-(I.25) to also solve the problem with an anisotropic permeable matrix. The effect of reducing  $\lambda_z$  with a constant  $\lambda_y$  is to reduce the effect of axial heat transfer by conduction (to increase Pe). This, as was shown for a channel with a uniform insert, leads to a decrease in heat-transfer rate on the initial thermal section.

Of thermal interest is the value of the ratio  $\alpha_{an}/\alpha$ , characterizing the change in the heat-transfer rate with the replacement of a uniform porous insert with a thermal conductivity  $\lambda_y$  by an anisotropic insert with a thermal conductivity  $\lambda_y$  and  $\lambda_z$ , other conditions being equal:

$$\frac{\alpha_{\rm an}}{\alpha} = \frac{Nu(\xi\Lambda, Pe\Lambda, Bi)}{Nu(\xi, Pe, Bi)} = \frac{Nu(\xi/Pe, Pe\Lambda, Bi)}{Nu(\xi/Pe, Pe, Bi)}.$$
(1.38)

This expression is easily analyzed quantitatively by means of the data shown in Fig. I.2. Considering that in the present case  $\xi_1/\text{Pe}_1 = \xi/\text{Pe}$ , we can see from (I.38) that  $\alpha_{an}/\alpha$  is nothing more than the ratio of the values of Nu for the same abscissa  $\xi/\text{Pe}$  with two dependences: PeA and Pe. Since  $\Lambda > 1$ , it follows that  $\alpha_{an}/\alpha$  is always less than unity and approaches unity when  $\xi \geqslant \xi_i$  or when Pe is large (Pe  $\rightarrow$  100). Thus, even a very substantial decrease in longitudinal thermal conductivity  $\lambda_z$  does not in the case of a constant transverse thermal conductivity  $\lambda_y$  reduce the heat-transfer rate if the length of the porous insert  $1/\delta$  is greater than  $\xi_i$  or if Pe is quite large (Pe  $\rightarrow$  100).

Similar results are obtained with a constant external heat flow.

Comparison with Experimental Data. Figure I.6 compares experimental data and values calculated with Eq. (I.32) for the mean heat-transfer criterion Nu for a plane channel with a porous filler in the case of a constant external heat flow.

The experimental unit and method of measurement were described in [7]. An annular channel was filled with a porous netted metal, the plane of the net having been normal to the long axis of the channel. Water and gaseous nitrogen were used as the coolant. The characteristics of the porous inserts are shown in Table 2. The mass rates of the coolant corresponded to the following ranges of Reynolds numbers calculated for a channel without a filler Re =  $G\delta/\mu$ : Re = 25-340, 1050-8600 for water; Re = 930-9100 for nitrogen. The experimental parameters pertain to the range in which the anisotropy of the thermal conductivity of the permeable matrix does not affect heat-transfer rate: the length of the porous insert was close to the length of the initial section either in the case of substantial values of the number Pe (Pe  $\approx$  100 or more) or at Pe < 10.



Fig. I.6. Comparison of calculated and experimental data on heat transfer in a channel with a porous filler. Characteristics of the specimens are shown in Table 2. The coolants were water (a) and gaseous nitrogen (b).

Comparison of the analytical and experimental results shows that they agree satisfactorily for different specimens and coolants. It should be noted that the empirically established increase in heat-transfer rate in channels with a filler compared to hollow channels reached a factor of 25-40 for water and 200-400 for nitrogen under the conditions investigated.

## LITERATURE CITED

- 1. U. R. Jaeger, Heat Exchanger, U.S. Patent No. 3289756 (1966).
- 2. F. A. Burne and R. V. Date, Concentric Tube Heat Exchanger with Sintered Metal Matrix, U.S. Patent No. 3323586 (1967).
- 3. F. A. Burne and E. I. Valyi, Modular Units and Use Thereof in Heat Exchangers, U.S. Patent No. 3415316 (1968).
- 4. R. B. Fleming, Heat Exchanger of Porous Metal, U.S. Patent No. 3433299 (1969).
- 5. A. Fortini and J. M. Kazaroff, Heat Exchanger and Method of Making, U.S. Patent No. 4245469 (1981).
- 6. Megerlin, Murphy, and Bergles, "Intensification of heat transfer in tubes by means of netted and brush inserts," Teploperedacha Ser. A, 96, No. 2, 30-38 (1974).
- 7. V. M. Polyaev, L. L. Morozova, E. V. Kharybin, and N. I. Avraamov, "Intensification of heat transfer in an annular channel," Izv. Vyssh. Uchebn. Zaved., Mashinostr., No. 2, 86-90 (1976).
- 8. V. V. Apollonov, A. I. Barchukov, V. I. Borodin, et al., "Increase in the thresholds for optical failure of metallic mirror surfaces during their cooling through a structure with open porosity," Pis'ma Zh. Tekh. Fiz., 4, No. 19, 1193-1197 (1978).
- 9. S. V. Belov, Porous Metals in Machine Construction [in Russian], Mashinostroenie, Moscow (1981).
- 10. V. A. Maiorov, "Flow and heat transfer of a single-phase heat carrier in porous metalceramic materials," Teploenergetika, No. 1, 64-70 (1978).
- 11. V. A. Maiorov, "Thermal conductivity of porous metals," in: Heat and Mass Transfer in Systems with Porous Elements [in Russian], ITMO Akad. Nauk Belorussian SSR, Minsk (1981), pp. 121-130.
- 12. Koch and Koloni, "Analysis of cooling efficiency for the case of coolant flow in a channel with a porous material," Teploperedacha Ser. C, 96, No. 3, 66-74 (1974).
- L. Pust, "Investigation of temperature fields in heat exchangers of porous cylindrical boards," Int. J. Heat Mass Transfer, 20, No. 11, 1255-1257 (1977).
   V. V. Apollonov, P. I. Bystrov, A. M. Prokhorov, et al., "Prospects for the use of porous structures to cool mechanical-optical elements," Kvantovaya Elektron., 6, No. 12, 2533-2545 (1979).
- 15. G. M. Kazakov and N. A. Kazakova, "General boundary-value problem for the temperature field of phases moving relative to each other," Teplofiz. Vys. Temp., 18, No. 6, 1221-1225 (1980).

- 16. M. M. Levitan and O. G. Rasin, "Efficiency of porous materials for intensifying heat exchange in channels," in: Heat and Mass Transfer in Systems with Porous Elements [in Russian], ITMO Akad. Nauk Belorussian SSR, Minsk (1981), pp. 91-99.
- 17. V. M. Polyaev and V. A. Maiorov, "Heat exchange in a channel with a porous insert in a forced-cooling system," Izv. Vyssh. Uchebn. Zaved., Mashinostr., No. 7, 51-55 (1982).
- 18. S. N. Bugorskaya, Yu. A. Zeigarnik, and V. K. Shikov, "Certain features of the formulation of heat-transfer problems with forced single-phase convection in porous layers," in: Convective Heat Exchange. Methods and Results of Investigations [in Russian], IVTAN, Moscow (1982), pp. 108-125.
- 19. S. N. Bugorskaya, Yu. A. Zeigarnik, B. S. Petukhov, et al., "Heat exchange with forced convection of single-phase heat carriers in a porous layer under conditions of unidirectional heating," in: Heat Exchange and Thermal Strains in Cooled Multilayered Systems [in Russian], IVTAN, Moscow (1982), pp. 65-90.

HEAT TRANSFER AND CRITICAL HEAT FLUXES IN THE BOILING OF AQUEOUS SOLUTIONS OF POLYETHYLENE OXIDE AT REDUCED PRESSURES UNDER NATURAL-CONVECTION CONDITIONS

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Experimental data are presented on heat transfer and critical heat fluxes in the boiling of aqueous solutions of polyethylene oxide of different concentrations under conditions of natural convection at atmospheric and reduced temperatures.

Study of the effect of polymeric additives on the heat-transfer rate during boiling is of both scientific and practical interest. Thus, it was shown in [1, 2] that the addition of a small quantity of a polymer to a heat carrier may lead to an increase in the heat-transfer coefficient during boiling. However, there has as yet been relatively little research in this area [1-6]. The investigation [3] studied the separation diameters and frequency of separation of bubbles at a single artificial vaporation center under conditions of natural convection and in a flow. Experiments were conducted in [4] on the boiling of aqueous solutions of polymers with forced flow. The experimental data reported in [1-4] on the boiling of polymer solutions was obtained only at atmospheric pressure and in a narrow range of heat fluxes. The exception is the work [6], which studied the effect of surfactants on heat transfer during the boiling of water at atmospheric and increased pressures in the region of subcritical heat fluxes. There is no data on critical heat fluxes in polymer solutions, and no study has been made of boiling in the region of reduced pressures and the effect of subheating of the liquid mass to the saturation temperature.

This article presents results of a complex of studies on heat exchange in aqueous solutions of polyethylene oxide (PEO) (molecular weight  $(3-5)\cdot10^6$ ), including experiments on heat transfer during saturated nucleate boiling and heat-transfer crises in saturated and subheated liquid under conditions of natural convection at atmospheric and reduced pressures. The study was performed with solutions with the following mass concentrations at 20°C: 0.002; 0.005; 0.01; 0.02; 0.04; 0.08; 0.16; 0.32; 0.64; 1.28%. The working section was a 2.5-mm-diameter stainless steel tube with a surface corresponding to a class six finish. The section was placed horizontally in the working volume and heated directly by an alternating current. A Chromel-Alumel thermocouple was placed inside the tube. In determining the temperature of the heating surface, we introduced a correction for the temperature drop in the wall. Before measurements were made, the heat-liberating surface of the section was used for 2-3 h at near-critical heat fluxes. Polyethylene oxide belongs to a class of polymers having the property of reverse solubility, which amounts to a deterioration in solubility with an increase in temperature. The heating of solutions with a concentration above 0.01% to 90-100°C was accompanied by turbidity and the precipitation of fine flocs.

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